

MATHEMATICS

CURRICULUM GUIDE

SEPTEMBER 1984

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MATHEMATICS

31

CURRICULUM GUIDE SEPTEMBER 1984

PROGRAM IMPLEMENTATION

Mandatory Implementation

Effective the 1984-85 school term, a revised Mathematics 31 course has been approved for mandatory implementation.

Change in Prerequisite Status

Effective the 1985-86 school term, Mathematics 30 will form the prerequisite for Mathematics 31. Students will be permitted to take Mathematics 30 and Mathematics 31 concurrently.

Concurrent Registration

Since students may be registered in Mathematics 30 and Mathematics 31 in the same semester or non-semester term coordination of topics in the two courses is recommended. The topics of Limits and Polynomials should be studied early in Mathematics 30 and the unit on Trigonometry in Mathematics 30 should be studied prior to Mathematics 31 students taking the section on Vectors.

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NOTE: THIS PUBLICATION IS A SERVICE DOCUMENT. The advice and direction offered is suggestive except where it duplicates or paraphrases the contents of the Program of Studies.

ACKNOWLEDGMENTS

Alberta Education acknowledges with appreciation the contribution of the following members of the Mathematics 31 Curriculum Committee operating under the direction of the Mathematics Curriculum Coordinating Committee and the Curriculum Policies Committee.

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ORIENTATION TO THE PROGRAM

CORE/ELECTIVE FORMAT

The Mathematics 31 course outline has been prepared to satisfy a core/elective format. The present core includes the majority of topics outlined in the 1975 Curriculum Guide. The major areas that have been deleted from the 1975 curriculum are Systems of Linear Equations, Matrices and Linear Transformations.

The graphing unit in Calculus has been defined so that it is no longer necessary to graph functions with discontinuities and/or cusps. Teachers continuing to teach any of these topics as part of the course are teaching an elective topic.

The core objectives as defined, are a prescribed portion of the course and should constitute 80% of the available teaching time. Although teachers have the freedom as to which elective(s) to teach, it should be noted that the elective component is a prescribed and mandatory part of the curriculum and constitutes the remaining 20% of the teaching time available.

COMMENT/APPLICATION SECTIONS

To supplement the objectives, comment/application sections have been included to provide some guidance for teachers presenting the program. Some examples have been included to suggest possible teaching strategies, to define the depth of an objective, or to show possible alternate solutions. Reference to the contribution of mathematics in the development of the human enterprise has also been made.

PRIORITY LEVELS

Each of the objectives in the core component has been assigned a priority level A, B, or C. These refer respectively to high, medium, and low priority. An assigned priority level does not indicate that an objective may or may not be taught, but rather to show the degree of emphasis to be placed on that objective. The priority level is also intended to give the teacher some guidance as to where the emphasis should be placed in evaluating the student's achievement in the program. Generally, objectives dealing with applications have been assigned an A priority, whereas those involving maintenance skills, notation, and definitions have been given a B, or C priority rating.

HISTORICAL PERSPECTIVE

Arnold Toynbee stated:

... at about the age of sixteen, I was offered a choice which, in retrospect, I can see that I was not mature enough, at the time, to make wisely. This choice was between starting on the calculus and, alternatively giving up mathematics altogether and spending the time saved from it on reading Latin and Greek literature more widely. I chose to give up mathematics, and I have lived to regret this keenly after it has become too late to repair my mistake. The calculus, even a taste of it, would have given me an important and illuminating additional outlook on the Universe, whereas, by the time at which the choice was presented to me, I had already got far enough in Latin and Greek to have been able to go farther with them unaided. So the choice that I made was the wrong one, yet it was natural that I should choose as I did. I was not good at mathematics; I did not like the stuff.... Looking back, I feel sure that I ought not to have been offered the choice; the rudiments, at least, of the calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to have to live. I was going to have to live in the Western World...and the calculus, like the full-rigged sailing ship, is...one of the characteristic expressions of the modern Western genius.¹

The concepts of limit, derivative, integral and vectors, which form the basis of the Mathematics 31 program, are a logical extension of algebraic skills developed in other high school mathematics programs. The concepts mentioned above should be taught in such a way that the contribution that famous mathematicians such as Eudoxus (403 - 355 BC), Archimedes (287 - 212 BC), Kepler (1571 - 1630), Fermat (1601 - 1665), Newton (1642 - 1727), Leibniz (1646 - 1716), Cauchy (1789 - 1857), Grassmann (1809 - 1877), Gibbs (1839 - 1903), Einstein (1879 - 1955) and many others have made to the development of mathematics and the resulting advancement of the human enterprise becomes known to students.

Reference to many mathematicians is made in the comments/applications section of this curriculum guide and should be included in the teaching of the Calculus and Vectors objectives. A much more detailed investigation of the contributions of one or more of these mathematicians might be undertaken as an elective topic.

¹ Arnold Toynbee, *Experiences*, Oxford University Press, pp. 12-13, 1969.

Students should become aware of the fact that Calculus has played a critical role in the development of science and thought since its conception over three hundred years ago. Calculus, as formally developed by Newton and Leibniz, was based upon the work of mathematicians who lived in the two thousand year period before them and was first used effectively in the universal analysis of functions to solve the problem of the motion of the moon and optimization problems. In the 19th century, Cauchy extended earlier work by developing a rigorous definition of limit. It is upon this background that Calculus has found application in science, engineering, industry and commerce.

The work of Gibbs in Vector Analysis appears as the study of geometric vectors while Grassmann's work in algebraic vectors provided the theoretical basis for considering more than three dimensions. Grassmann's work in the area of tensor calculus was used by Einstein in his theory of relativity.

By combining the knowledge bases of vectors, calculus and physics it is possible to solve problems dealing with the orbits of celestial bodies, satellites and trajectories.

SUGGESTED TIME ALLOCATION

The percentages indicated below represent the proportion of time allotted to each topic of the course.

<u>Topic</u>	<u>% of Total Time</u>	
1. Calculus (50%)		
A. Introduction to Calculus	8%	} 38%
B. Derivatives of Algebraic Relations	12%	
C. Applications of Derivatives	18%	
D. Integration		12%
2. Vectors (30%)		
A. Geometric Vectors and Their Application		11%
B. Algebraic Vectors and Their Application		13%
C. Dot Product		6%
3. Electives (20%) *		
A. Volumes of Revolution		7%
B. Calculus of Logarithmic Functions		13%
C. Trigonometric Extensions		7%
D. Calculus of Trigonometric Functions **		13%
E. Systems of Linear Equations		7%
F. Matrices and Linear Transformations		13%
G. Polar Coordinates		7%

* Teachers may choose any combination from the list of electives in order to account for about 20% of the available class time. Please refer to the discussion of the Elective Component on page 12 of this Guide.

** This elective should be studied only if Elective C has been completed.

PLANNING SUGGESTIONS

Suggested time allocations indicated earlier will not be realized if a clear understanding of the objectives stated herein has not been achieved. The objective statements limit the scope of the topics of Mathematics 31. The teacher must be careful, when selecting questions, to be cognizant of the difficulty level of each exercise. Elective topics could be chosen to complement topics studied in Calculus or Vectors or they could be unrelated to Calculus or Vectors. Several suggestions related to the contribution of Calculus and Vectors to the advancement of the human enterprise have been included in the Comments/Applications section of the curriculum guide and should be discussed with students.





LEARNING RESOURCES

Prescribed Texts:

Math 31

H.A. Elliott, K.D. Fryer, J.C. Gardner,
Norman J. Hill
Toronto: Holt, Rinehart & Winston of
Canada, Limited, 1984.

or

Calculus, Complex Numbers and Polar Coordinates

H.A. Elliott, K.D. Fryer, J.C. Gardner,
Norman J. Hill
Toronto: Holt, Rinehart & Winston of
Canada, Limited, 1980.

and

Vectors, Matrices and Algebraic Structures

H.A. Elliott, K.D. Fryer, J.C. Gardner,
Norman J. Hill
Toronto: Holt, Rinehart & Winston of
Canada, Limited, 1980.

Recommended Learning Resource

Computer Graphing Experiments Vol. 1
Computer Graphing Experiments Vol. 2
Computer Graphing Experiments Vol. 3

Don Mills: Addison Wesley (Canada)
Limited, 1981.

Additional Teacher References

See Appendix C.

GOALS OF THE SENIOR HIGH SCHOOL MATHEMATICS PROGRAM

Although the different courses of the senior high school mathematics program have different specific objectives, the goals of the senior high mathematics program are set forth in relation to three main expectations and needs: those of the individual, those of the discipline of mathematics and those of society at large. They are listed as follows:

Student Development

- a) To develop in each student a positive attitude towards mathematics.
- b) To develop an appreciation of the contribution of mathematics to the progress of civilization.
- c) To develop the ability to utilize mathematical concepts, skills and processes.
- d) To develop the powers of logical analysis and inquiry.
- e) To develop an ability to communicate mathematical ideas clearly and correctly to others.

Discipline of Mathematics

- a) To provide an understanding that mathematics is a language using carefully defined terms and concise symbolic representations.
- b) To provide an understanding of the concepts, skills and processes of mathematics.
- c) To provide an understanding of the common unifying structure in mathematics.
- d) To furnish a mode of reasoning and problem-solving with a capability of using mathematics and mathematical reasoning in practical situations.

Societal Needs

- a) To develop a mathematical competence in students in order to function as citizens in today's society.
- b) To develop an appreciation of the importance and relevance of mathematics as part of the cultural heritage that assists people to utilize relationships that influence their environment.
- c) To develop an appreciation of the role of mathematics in man's total environment.

OBJECTIVES OF THE MATHEMATICS 31 PROGRAM

The general objectives of the high school mathematics program stated in the previous section apply to Mathematics 31. Following is a statement of the more specific objectives of the course:

1. To develop an understanding of the properties of functions and their role in many different areas of mathematics.
2. To increase the student's awareness of the scope of mathematics.
3. To prepare the student for further courses in calculus, vector analysis and related fields.
4. To develop skills that will increase a student's ability to simplify algebraic and numeric expressions.
5. To assist students to see that the mathematics of calculus and vectors has many practical applications.

PROBLEM SOLVING

The discipline of mathematics provides one of the best opportunities to practise the process of problem-solving. The techniques learned and experience gained are transferable to almost any problem-solving situation and provide the basis of successful living.

Everyone's job involves problem-solving. Although many of the problems we have to contend with are repetitious and are easily resolved once the initial solution is known, the most challenging and exciting aspects deal with problems that are completely new to the individual or assist in arriving at solutions in new and creative ways.

A primary purpose of education is to provide a student with the tools to solve a wide variety of problems. Specific problem-solving skills can be taught; however, the methods of successfully tackling untried problems are not always fully understood. There are stages which one may follow, such as:

- a) Fully understanding the problem.
- b) Accepting the challenge of searching for a solution.
- c) Making a reasonable guess at the answer (this provides a check when a solution is finally found).
- d) Pictorially or graphically representing the important aspects of the problem if this is at all possible.
- e) Solving a simpler version of the problem or a special case of the more general problem.
- f) Using specific cases to discover patterns or different approaches to the problem.
- g) Solving the general problem.
- h) Checking the solution in the special cases solved before.
(Does the solution seem reasonable based on your initial guess?)

The above stages are helpful in tackling any problem. Considerable intuition and experience are still necessary to tackle the really difficult problems.

The strategies of problem solving are discussed in greater detail in Appendix A.

APPLICATIONS

A central and encompassing approach to making the study of Mathematics 31 both meaningful and interesting is to place a major emphasis in the area of applications. An application is the process of using a mathematical skill to arrive at a solution to a real life or practical situation. Applications should incorporate interesting, useful, relevant and diverse examples from real-life situations. The significance of using applications is based on relating mathematical concepts and skills to problems encountered in society and the environment. Mathematics may be related to countless aspects of living. Our task is to have students recognize these relations, develop an understanding of the interrelationships that exist within topics of the Mathematics 31 program and then learn to transfer their use of mathematics to other situations.

Applications, like problem-solving, should be integrated into the overall program rather than be dealt with as an independent unit. Whenever possible, integration and coordination with other subject areas should be encouraged. When applications require extensive computations, the use of the calculator or a computer may become a necessary component of the learning process to avoid time-consuming calculations.



ELECTIVE COMPONENT

The elective component is designed to be an interesting motivational and challenging aspect of the mathematics program. The outlines for the elective topics are intended to act as guidelines for teachers. Teachers may choose to:

- a) Follow the suggested elective objectives.
- b) Select a sequential subset of objectives from one or more elective units.
- c) Develop their own ideas for incorporation in the elective component.

The additional references provide useful material for many of the suggested elective units.

A. Structure of the Elective

The elective component of the program may be:

1. A content area not prescribed as core.
 2. A locally developed unit as determined by the teacher or school system.
- OR
3. An extension of the subject matter in any of the core topics to provide students with enrichment.

B. Guidelines

Teachers should keep the following guidelines in mind:

1. The list of topics (page 4) is open-ended so that the interests and abilities of students may be taken into account.
2. Student initiated projects may be considered as an elective.
3. A teacher should make use of any appropriate resources.
4. Electives should be included in the course throughout the year, wherever appropriate. Where more than one elective is included, equal time does not necessarily have to be assigned to each.

5. The elective component of the program should be included in the evaluation of the students.
6. Prerequisite core material may be required before some electives are attempted.

C. Suggested Time Allotments

The time allocated to topics in the elective component of the course may vary according to the topic(s) chosen and instructional preferences of the teacher. Twenty percent of the instructional time is to be devoted to cover an elective topic or, alternatively, two or more topics.

D. Provision for the Academically Talented

Topics within the elective component of the course should be utilized to challenge the academically talented students. Teachers may also extend core topics and concepts to a higher level of complexity to meet the needs of the stronger academic student. However, the intent of the elective portion of Mathematics 31 is to increase the student's awareness of the scope of mathematics, not merely to provide extra depth in a single topic.



CORE OBJECTIVES



CALCULUS

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
A. <u>Introduction to Calculus</u>	After Euclidean geometry, the greatest creation of all of mathematics was the calculus. It was not developed primarily to answer many of the problems of the ancient Greek mathematicians, but rather in response to the needs created by the scientific endeavors of the 17th century. The teacher may wish to have the students research the development of mathematics during this crucial time. Special reference may be made to Descartes, Pascal, Fermat, Kepler, Leibniz, Newton, and Cauchy.			
1. Maintain previously developed skills in finding an equation of a line.	Consideration should be given to: 1. oblique lines 2. lines parallel to the axes 3. conditions for parallel and perpendicular lines	C	P. 1-3	P. 1-3
2. Define secant line.		C	P. 3	P. 3
3. Determine the equation of a secant line.		B	P. 4-5 Review P. 23 # 2	P. 4-5 Review P. 23 # 2
4. Define tangent line.	Reference may be made to secant lines where the distance between the two points of intersection approaches zero.	C	P. 5-7	P. 5-7

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
5. Define the limit of an infinite sequence.	The concept of limit was first dealt with by the Greek mathematicians Eudoxus and Archimedes who were concerned with finding the area of a circle by circumscribing and inscribing regular polygons thus trapping the area of the circle between the two polygons. Two thousand years later, a more formal structure was given to the limit by problems arising out of Kepler's work. At first, the notion of indivisibles was suggested by Cavalieri, Wallis and Roberval. Later, Fermat, Newton, Leibniz and Cauchy formalized a more rigorous treatment of limit.	B	P. 64	P. 64
6. Determine the limit of an algebraic function as the independent variable approaches a fixed value.	Discussion must include a fixed value where the function is not defined. Example: $\lim_{x \rightarrow -3} \left[\frac{x^3 + 27}{x + 3} \right]$ An extension of this occurs when the independent variable approaches infinity (increases without bound). Example: $\lim_{x \rightarrow \infty} \left[\frac{x^2 + 7}{4x^2 + 1} \right] = \frac{1}{4}$ As an extension of this objective, show that a function differs from its limit by less than a given value when the independent variable has a given domain. Rigorous treatment is not necessary. Properties of absolute value should be reviewed or taught as required.	A	P. 64 # 1-10 P. 70-73	P. 64 # 1-10 P. 70-73
			P. 64 # 11-17 P. 73	P. 64 # 11-17 P. 73

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
7. Verify the following limit theorems: 7.1 $\lim_{x \rightarrow a} k = k$, k is a constant 7.2 $\lim_{x \rightarrow a} f(x) \pm g(x) = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x)$ 7.3 $\lim_{x \rightarrow a} [f(x)g(x)] = \left[\lim_{x \rightarrow a} f(x) \right] \left[\lim_{x \rightarrow a} g(x) \right]$ 7.4 $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ (if $\lim_{x \rightarrow a} g(x) \neq 0$) 7.5 $\lim_{x \rightarrow a} k [f(x)] = k \left[\lim_{x \rightarrow a} f(x) \right]$		B	P. 75-77	P. 75-77
<u>B. Derivatives of Algebraic Relations</u>				
1. Define the derivative of a function in terms of limits.	$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$	B	P. 74-75	P. 74-75
2. Become familiar with a variety of derivative notations.	$D_x y = \frac{dy}{dx} = f'(x) = y'$	C	P. 74-75	P. 74-75
3. Develop the derivatives of the following functions from first principles using limits: 3.1 linear 3.2 quadratic 3.3 cubic 3.4 reciprocal	Examples: $y = 3x$ $y = 4x^2$ $y = 2x^3$ $y = \frac{5}{x}$ In addition to these, examples should include more than one term.	A	P. 12-19	P. 12-19

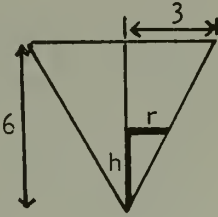
OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
4. Determine the derivatives of algebraic functions by generalizing the results obtained from first principles.	Example: $y = x^3 + 2x^2 - 3x + 1$	A	P. 17 #3 Review P. 23 #4	P. 17 #3 Review P. 23 #4
5. Determine the slopes of tangent lines to algebraic functions.		A	P. 8-9	P. 8-9
6. Determine the equations of tangent lines to algebraic functions.		A	P. 20- 22	P. 20- 22
7. Derive the: 7.1 power rule 7.2 product rule 7.3 quotient rule	If $y = x^n$, then $y' = nx^{n-1}$. Examples: $y = (x^3-1)(1-3x^2)$ $y = \frac{x^3-1}{x^2+x}$	B	P. 79- 83 P. 89- 95	P. 79- 83 P. 89- 95
	Students could be expected to derive these rules where "n" is a member of the natural numbers but applications should include rational numbers.			
8. Apply the chain rule, power rule, product rule and quotient rule.	The derivation of the chain rule could be used as a teaching strategy. However, the student should not be expected to derive the rule since the derivation may be too rigorous. Students should be expected to perform differentiations using the chain rule in combination with the product or quotient rules. (Continued on next page.)	A	P. 87- 88 Review P. 96 Omit 25-30	P. 87- 88 Review P. 96 Omit 25-30

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
8. (Continued)	Examples: $y = (3x^2 + 1)^5$ $y = (1 - 2x^3)$ $y = \sqrt{3x^2 + 1}$ $y = (3x^2 + 1)^4 (2x^2 + 5)$ $y = \frac{(2x^5 + 2)^3}{3}$			
9. Determine the derivatives of relations.	Use the technique of implicit differentiation. (Variables in addition to x and y should also be used).	A	P. 97-105	P. 97-105
10. Determine the equations of tangent lines to relations.		A	P. 100-105	P. 100-105
C. <u>Applications of Derivatives</u>	While working with the theory of light, in the middle of the 18th century Pierre Maupertuis announced the Principle of Least Action. Although he advocated his principle for theological reasons, modern day mathematicians accept the minimizing or maximizing processes because they are aesthetically appealing and scientifically helpful. It has been shown that nearly all gravitational, optical, dynamical, and electrical laws can be obtained by maximizing or minimizing a function as the time integral of kinetic potential. Albert Einstein showed that maximization of a function, called the interval, accounts for the observed path of the planets.			

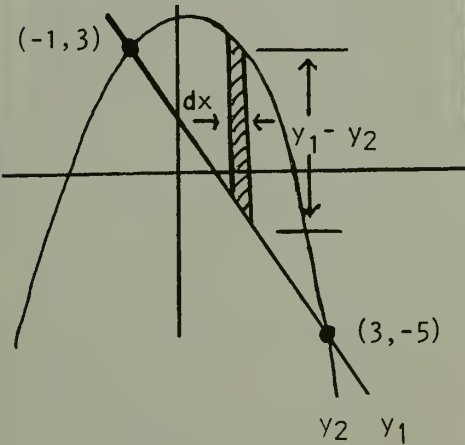
OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT	REF.
			1980 Edition	1984 Edition
1. <u>Maxima and Minima</u>				
1.1 Maintain previous skills in solving polynomial equations.	Students should be able to identify polynomials, determine zeros of polynomials, and factor polynomials.	C	P. 69-70	P. 69-70
1.2 Identify the domains for which the first derivative is greater than zero, less than zero or equal to zero.	Reference could be made to the tangent to a curve rising to the right or left, or parallel to the x-axis in order to determine maxima or minima.	B	P. 49-52	P. 49-52
1.3 Define and identify maxima and minima using first and second derivatives.	The use of the graphs of $f(x)$, $f'(x)$ and $f''(x)$ is an effective teaching strategy to justifying the tests for maxima and minima.	A	P. 49-52 P. 113-116	P. 49-52 P. 113-116
1.4 Identify formulae for perimeter, area and volume of various plane and solid figures.	Formula for circles, quadrilaterals, spheres, cones, rectangular solids, triangles and cylinders should be mentioned.	C	P. 53-54 P. 131-135	P. 53-54 P. 131-135
1.5 Solve applied problems.	A review of maxima and minima from the standpoint of quadratic functions might be useful with new solutions developed using differentiation. It is the intent that students quickly proceed to questions beyond the scope of quadratic relations. Example: When a population of N bacteria is introduced into a nutrient medium the population grows according to the equation $P = N + \frac{Nt}{81+t^2}$ where t is the number of hours of growth. If $N = 1000$, determine the maximum size of this population and when that size is realized.	A	P. 55-60 P. 128-131	P. 55-60 P. 128-131

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
2. <u>Graphing</u>				
2.1 Maintain previous skills of finding x- and y-intercepts of functions.	Choose polynomials that have integral intercepts.	C	P. 105-113	P. 105-113
2.2 Find the coordinates of any point where a relative maximum or minimum occurs.		B	P. 105-113	P. 105-113
2.3 Define point of inflection.	Points of inflection indicate a change of concavity.	B	P. 113	P. 113
2.4 Find the coordinates of any points of inflection.		A	P. 113-116	P. 113-116
2.5 Sketch graphs of polynomial functions identifying x- and y-intercepts, relative maxima, minima and any points of inflection.	Examples: $y = 2x^3 + 3x^2 + 1$ $y = 3x^4 + 4x^3 - 12x^2$ The objective may be extended to include relations with discontinuities, asymptotes or cusps. This is beyond the scope of the objective	A	P. 113-119	P. 113-119
3. <u>Motion Problems</u>				
3.1 Define average velocity and solve related problems.	The concept of acceleration had been known to Nicole Oresme (1323-1382) but did not play a significant role in dynamics until the 17th century.	B	P. 25-27	P. 25-27
3.2 Define instantaneous velocity using limits and derivatives.		B	P. 27-36	P. 27-36
3.3 Define acceleration and solve related problems.		C	P. 36-39	P. 36-39

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
3.4 Define instantaneous acceleration using limits and derivatives.		B	P. 36-39	P. 36-39
3.5 Solve applied motion problems using differentiation.		A	P. 35-36 P. 39 P. 41	P. 35-36 P. 39 P. 41
<u>4. Related Rate Problems</u>				
4.1 Use the chain rule for the differentiation of a function with respect to a variable not in the function.	Example: $A = \pi r^2$ $D_t A = 2\pi r D_t r$ Notation may vary depending on text.	B	P. 121-127	P. 121-127
4.2 Interpret a derivative as a rate of change of one variable with respect to another.	A derivative may be interpreted as more than just a slope. Application may extend to topics of velocity ($D_t S$) acceleration ($D_t V$), and change of volume ($D_r V$, $D_A V$, $D_t V$) etc.	B	P. 128-136	P. 128-136
4.3 Solve applied problems using rates.	Express the function in one variable and then differentiate. Example: A conical hopper is being filled with wheat at the rate of $0.5\text{m}^3/\text{s}$. The hopper is 6m high and 3m in radius at the top. Find the rate at which wheat is rising in the hopper when the depth is 2m. (Continued on next page.)	A	P. 124-127	P. 124-127

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
4.3 (Continued)	$\frac{h}{r} = \frac{6}{3}$ $\therefore r = \frac{h}{2}$ $V = \frac{1}{3} \pi r^2 h$ $V = \frac{1}{3} \pi \left(\frac{h^2}{4} \right) h$ $V = \frac{\pi h^3}{12}$ $\therefore D_t V = \frac{3\pi h^2 D_t h}{12}$ $\frac{1}{2} = \frac{3\pi (4) D_t h}{12}$ $\frac{1}{2\pi} = D_t h$ 			
	The wheat is rising at the rate of $\frac{1}{2\pi}$ m/s when the depth is 2m.			
D. <u>Integration</u>	It is interesting to note that the foundations for integral calculus began nearly 2000 years before the 17th century development of differential calculus. The teacher and student may wish to research the developments from the time of Eudoxus (408-355 B.C.) and Archimedes (287-212 B.C.) to the perfection of the integration process by Pierre de Fermat (1601-1665), Gottfried Leibniz (1646-1716), Jakob Bernoulli (1654-1705), and later Augustin Louis Cauchy (1789-1857).			

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
1. Define integration.	Integration is the inverse process of differentiation. The end result of differentiation is the derivative. The end result of integration is the antiderivative or indefinite integral.	B	P. 138	P. 138
2. Find the family of curves whose first derivative has been given.	Limit the questions to polynomial functions and simple rational algebraic functions. Examples: $\frac{dy}{dx} = x^{\frac{1}{2}} + \frac{1}{x^2}$ $\frac{dy}{dx} = 3x^3 - 7x + 12$	B	P. 139-145 P. 165-168	P. 139-145 P. 165-168
3. Find a specific member of a family of curves given the derivative and the coordinates of a point on a curve.		A	P. 142-145	P. 142-145
4. Solve applied motion problems.	Example: A model rocket is powered by a solid-fuel motor which delivers an acceleration to the rocket given by $a = t^2 + 18t$ where time t is measured in seconds and acceleration a is measured in m/s^2 . Find the velocity and height of the rocket at burnout.	A	P. 145-150	P. 145-150
5. Define and evaluate definite integrals.		A	P. 159-165	P. 159-165

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
6. Determine the area between a curve and the x-axis over a given domain. 6.1 If $f(x)$ has a constant sign in a given domain. 6.2 If $f(x)$ changes sign in a given domain.	Relate the value of the integral at $x = a$ to the area between the curve, the x axis and over the domain $0 \leq x \leq a$.	A	P. 150-156	P. 150-156
7. Determine the area between two curves over a given domain.		A	P. 169-175	P. 169-175
8. Determine the area between two intersecting curves.	One method that could be used is to divide the region into strips with elemental area $(y_1 - y_2)dx$ with consideration given to possible points where y_1 and y_2 cross in the given domain. Example: Find the area between $y_1 = -x^2 + 4$ and $y_2 = -2x + 1$  (Continued on next page.)	A	P. 169-175	P. 169-175

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
	STEP 1: Find the points of intersection and sketch the curves. STEP 2: From the diagram $y_1 > y_2$ between the points of intersection $\therefore$ Define $Y = y_1 - y_2$ $= (-x^2 + 4) - (-2x + 1)$ $= -x^2 + 2x + 3$ STEP 3: Integrate $Y = -x^2 + 2x + 3$ to obtain $F(x) = \frac{-1}{3} x^3 + x^2 + 3x$ STEP 4: Find the area by evaluating ${}_{-1}A_3 = F(3) - F(-1)$			

VECTORS

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
A. <u>Geometric Vectors and Their Application</u>	This section should include a discussion of problems in two- and three-dimensional space.			
1. Maintain previous skills in trigonometry.	Skills required are: - Basic definition of primary trigonometric ratios. - Law of Cosines. - Law of Sines.	C		
2. Define, sketch and apply three-dimensional geometric relationships.	Three-dimensional sketches are difficult to do effectively. Make use of various mathematical aids such as metre sticks, corners of rooms, plastic straws, pipe cleaners, computer courseware, etc. Teachers should make reference to: - Skew lines. - Intersection of lines and planes. - Intersection of two and/or three planes. - Definition of dihedral angle.	B	P. 1-5	P. 269-273
3. Define and/or illustrate: 3.1 geometric vectors 3.2 equivalent vectors 3.3 collinear and coplanar vectors (Continued on next page.)		B	P. 5-7 P. 26-27	P. 273-274 P. 294-295

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
3.4 vector addition 3.4.1 for collinear vectors 3.4.2 for non-collinear vectors 3.5 inverse vectors	Both the triangle and the parallelogram method should be used to illustrate vector addition. Use both notations: 1. The inverse of $\vec{u}$ is $-\vec{u}$. 2. The inverse of $\vec{AB}$ is $\vec{BA}$ or $-\vec{AB}$.		P. 7-13	P. 275- 281
3.6 scalar multiplication 3.7 vector subtraction	Define zero vector.		P. 26- 27	P. 294- 295
	Apply to forces in equilibrium as they relate to physics.			
4. Develop vector notation.	It should be noted that several texts use a variety of vector notation.	B	P. 6	P. 274
5. Illustrate graphically the following algebraic properties of vector addition: 5.1 commutative property 5.2 associative property 5.3 identity property 5.4 inverse property		C	P. 13- 17	P. 281- 285
6. Apply the algebraic properties of vector addition.	The main emphasis should be upon using these properties to achieve a single vector and not necessarily upon which of the properties have been used.	A	P. 13- 17	P. 281- 285

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
7. Apply the properties of multiplication by a scalar.	For the following properties, $\vec{u}$ and $\vec{v}$ are vectors and k and ℓ are real numbers. $k\vec{u}$ is a vector. $k\vec{u} = \vec{u}k$. - If $k = \ell$ and $\vec{u} = \vec{v}$, the $k\vec{u} = \ell\vec{v}$. $(k\ell)\vec{u} = k(\ell\vec{u})$. $1\vec{u} = \vec{u}$. $(-1)\vec{u} = -\vec{u}$. $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$. $(k + \ell)\vec{u} = k\vec{u} + \ell\vec{u}$. $k\vec{u} = k \vec{u}$	B	P. 26-31	P. 294-299
8. Apply vector addition to problems which include definition of resultant and equilibrant.	Problems should involve addition of vectors in geometric figures, vector forces and velocities. Attach importance to bearing notation as well as N.W., S.E. etc. Students should be required to find one force when given the resultant and another force. Consult Stolberg & Hill: <u>Fundamentals of Physics</u>. For numerous examples of force and resultant problems. (Continued on next page.)	A	P. 17-23	P. 285-291

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
8. (Continued)	Teachers may wish to discuss the use of vector addition for problems dealing with trajectories, orbits, and navigation. It should be pointed out that these problems become more complex in outer space when gravitational forces from other planetary bodies are in effect.			
9. Define and apply resolution of vectors into rectangular components.	This objective should be taken to the extent of using three-dimensional vectors.	A	P. 23-26	P. 291-294
10. Define and apply linear combinations of geometric vectors.	Students should be limited to forming linear combinations of geometric vectors in 2-space. Example: An alternate solution to Example 4, P.306 is: $\vec{AB} : \vec{BC} = -5 : 3$ $\vec{AB} = -\frac{5}{3} \vec{BC}$ $\vec{AO} + \vec{OB} = -\frac{5}{3} (\vec{BO} + \vec{OC})$ $3\vec{AO} + 3\vec{OB} = -5\vec{BO} - 5\vec{OC}$ $-3\vec{OA} + 3\vec{OB} = 5\vec{OB} - 5\vec{OC}$ $-2\vec{OB} = 3\vec{OA} - 5\vec{OC}$ $\vec{OB} = -\frac{3}{2} \vec{OA} + \frac{5}{2} \vec{OC}$	A	P. 31-40	P. 299-308

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
B. <u>Algebraic Vectors and Their Application</u>				
1. Maintain previous skills in plotting points and finding the distance between two points in a plane.		C	P. 73- 77	P. 341- 345
2. Plot points and find the distance between two points in a three-dimensional system.	Students should be exposed to the derivation of the distance formula in 3-space. Example:	A	P. 77- 79	P. 345- 347
	Plot the points A(1,2,3) and B(2,1,-1) and determine the distance from A to B.			
3. Define: 3.1 algebraic vectors 3.2 equivalent algebraic vectors 3.3 zero vectors 3.4 unit vectors	Care should be taken not to confuse (a,b) which is a point in a co-ordinate plane with $\vec{u} = (a,b)$ which is a vector whose initial point is at the origin and terminal point at (a,b).	B	P. 47- 48 P. 81	P. 315- 316 P. 349
4. Define the following concepts: 4.1 addition and subtraction of algebraic vectors 4.2 scalar multiplication of algebraic vectors		B	P. 48- 49	P. 316- 317

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
5. Solve equations involving addition and subtraction of algebraic vectors and scalar multiplication of algebraic vectors.	Example: Solve for a and b if: $2(a,4) - (5,b) = (a + 2, b - 5)$	A	P. 48-50	P. 316-318
6. Express two- and three-dimensional vectors in algebraic form.		B	P. 50-53	P. 318-321
7. Solve problems related to algebraic vectors, given initial points and terminal points.	Example: What algebraic vector corresponds to $\overrightarrow{PQ}$ if P has coordinates (2,3,4) and Q has coordinates (-1,3,11)?	B	P. 50-53	P. 318-321
8. Define collinearity of algebraic vectors in terms of scalar multiples.		B	P. 57	P. 325
9. Solve problems involving collinear algebraic vectors.	Example: Find the value of k so that the vectors (1,5,-3) and (-2,k,6) are collinear.	A	P. 57-58	P. 325-326
10. Define coplanar vectors in terms of linear combinations.	Students should be exposed to sketching a plane formed by two non-collinear vectors. Teachers may wish to extend this objective to a discussion of bases for 2-space and 3-space. This is beyond the scope of the objective.	B	P. 59-60	P. 327-328

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
11. Solve problems related to coplanar vectors.	Example: Is $\vec{u} = (1,1,9)$ in the plane determined by $\vec{v} = (2,1,4)$ and $\vec{w} = (3,1,-1)$?	A	P. 59-62	P. 327-330
12. Find the length of two- and three-dimensional vectors.	Teachers may wish to have students sketch vectors in 3-space with initial point at the origin or off the origin.	A	P. 79-81	P. 347-349
C. <u>Dot Product</u>				
1. Define dot product for algebraic vectors.	Dot product, inner product and scalar product can be used interchangeably.	B	P. 83-85	P. 351-353
2. Simplify and/or evaluate expressions using the dot product of algebraic vectors.		A	P. 83-85	P. 351-353
3. Verify that the dot product for geometric vectors is equivalent to the dot product of their algebraic equivalent.	Application may involve verifying that algebraic dot product is equivalent to the geometric dot product.	B	P. 90-92	P. 358-360
4. Solve problems involving the angle between vectors.	Problems should involve both 2- and 3-space.	A	P. 92-95	P. 360-363
5. Solve problems involving perpendicular (orthogonal) vectors.		A	P. 92-95	P. 360-363
6. Define the projection of one vector onto another.	These projections should be done using the dot product.	B	P. 96-99	P. 364-367

OBJECTIVE	COMMENT/APPLICATION	PRIORITY	TEXT REF.	
			1980 Edition	1984 Edition
7. Solve problems related to the projection of one vector onto another.	Apply to both 2-space and 3-space.	A	P. 96-99	P. 364-367
8. Resolve a vector into two perpendicular components, one of which is parallel to a given vector.	Apply to both 2-space and 3-space. The resolution of forces is a practical application.	A	P. 99-102	P. 367-370
9. Define work in terms of dot product and solve related problems.	Apply to both 2-space and 3-space. Consultation of physics texts is encouraged at this point. <u>Physics: Principles & Problems</u> by Murphy & Smoot is a source of problems. (pp. 180-181, #13, 14, 16, 17) In simplest terms, work can be thought of as force times distance. Questions usually assume all distances and forces to be constant. Scientists, however, have been confronted with problems where this is not the case. Students may like to discuss the following two situations: 1) How much work is done in taking an object with mass 90 kg and raising it 500 km above the earth? (Note that the force acting on the body changes with altitude.) 2) Suppose water fills a cylindrical barrel 1 m high and 1 m in diameter. How much work is done in pumping the water out of the barrel? (Note that every molecule of water will be raised through a different distance.) These problems are beyond the scope of this course but point students toward problems that may be solved using the concepts of calculus and vectors.	A	P. 102-105	P. 370-373

ELECTIVE OBJECTIVES



A. VOLUMES OF REVOLUTION

(7% elective)

Having discussed integration as a means to determine the area between curves over a given domain, it is natural to extend these ideas into a three-dimensional framework.

Prescribed References

1. "Calculus, Complex Numbers and Polar Coordinates"
Holt, Rinehart and Winston - 1980
2. "Math 31"
Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
1. Determine the volume of revolution of a given curve over a given domain.	It is suggested that a sketch is an integral part of the teaching strategy for these objectives.	P. 175	P. 175
2. Determine the volume of revolution between two curves over a given domain.		P. 180	P. 180

Additional References

1. Coxford, A.F., and J.N. Payne, HBJ Advanced Mathematics: A Preparation for Calculus, Harcourt Brace Jovanovich, New York, 1978.
ISBN 0-15-353940-2
2. Larson, R.E., and R.P. Hostetler, Calculus With Analytic Geometry, 2nd Edition, D.C. Heath and Company, Toronto.
ISBN 0-669-04530-6
3. Lynch, R.O., A First Course in Calculus Including Analytic Geometry, 3rd Edition, Ginn and Company, Lexington, Massachusetts.
ISBN 0-663-41967-0

B. THE CALCULUS OF LOGARITHMIC FUNCTIONS

(13% elective)

Special Note: This elective should be selected only if the students have previously covered the logarithmic unit in Math 30.

Having dealt with the differentiation and integration of the polynomial function, further application of these techniques may be made to e^x and $\ln(x)$. Care should be taken to avoid problems using the calculus of trigonometric functions unless that elective has been covered.

Prescribed Resources

1. "Calculus, Complex Numbers and Polar Coordinates"
Holt, Rinehart and Winston - 1980
2. "Math 31"
Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
1. Maintain previous logarithmic skills.	See objectives p. 81-82 of the Math 10-20-30 Curriculum Guide.		
2. Define e as a limit.	$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$; $e = 2.7182\dots$	P. 215	P. 215
3. Develop the derivative of the natural logarithmic function.	Derivation is encouraged but not necessarily to the extent of being tested. $y = \ln(x), x > 0$	P. 217- 218	P. 217- 218
4. Apply the derivative of the natural logarithmic function to determine the derivative of related logarithmic functions utilizing the chain, power, product and quotient rules.	It would be useful to use the rules for the laws of logarithmic functions when using the rules for derivatives (product rule, quotient rule). Example: $y = \ln(x^2 + 1)^{\frac{1}{2}}$ becomes $y = \frac{1}{2} \ln(x^2 + 1)$	P. 217- 220	P. 217- 220

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
5. Develop the derivative of $y = e^x$.	$y = e^x$ is commonly referred to as "THE" exponential function.	P. 221-222	P. 221-222
6. Apply the derivative of $y = e^x$ to determine the derivative of related exponential functions utilizing the chain, power, product and quotient rules.		P. 221-224	P. 221-224
7. Solve related maxima and minima problems.	Problems may include bacteria growth and radioactive decay. This is a natural cut-off point for a 10% elective unit.	P. 225-229	P. 225-229
8. Define the integral of $f = e^x$ and $y = 1/x$.		P. 230	P. 230
9. Determine the family of curves whose first derivative has been given.		P. 230-232	P. 230-232
10. Determine the area between a curve and the x-axis over a given domain.		P. 232-235	P. 232-235
11. Determine the area between two curves over a given domain.		P. 232-235	P. 232-235
12. Solve problems related to the equation $D_x y = ky$.	Notation may vary depending on text.	P. 235-238	P. 235-238

C. TRIGONOMETRIC EXTENSION

(7% elective)

Special Note:

This elective should be selected only if the students have previously covered the trigonometry unit in Mathematics 30. The objectives in this elective are not covered in the prescribed text. Teachers selecting this elective will have to refer to the additional references suggested.

OBJECTIVE	COMMENT	TRIGONOMETRY WITH TABLES	HBJ ADVANCED MATHEMATICS	CALCULUS WITH ANALYTIC GEOMETRY
1. Maintain previous trigonometric skills.	Particular emphasis should be given to the Sum and Difference Identities for sine, cosine and tangent functions.	P. 201-205	P. 141-149	Appendix A1-13
2. Derive the Double- and Half-Angle Identities.	1. $\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$ 2. $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$ $= 2 \cos^2(\theta) - 1$ $= 1 - 2 \sin^2(\theta)$ 3. $\sin\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 - \cos(\theta)}{2}}$ 4. $\cos\left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1 + \cos(\theta)}{2}}$	P. 206-208	P. 152-154	Appendix A14
3. Apply the Double and Half-Angle formulae to prove identities and to solve equations.		P. 209 P. 215-217	P. 155	Appendix A25
4. Derive the Sum and Product Identities.	1. $\sin(A+B) + \sin(A-B) = 2 \sin(A) \cos(B)$ 2. $\sin(A+B) - \sin(A-B) = 2 \cos(A) \sin(B)$ 3. $\cos(A+B) + \cos(A-B) = 2 \cos(A) \cos(B)$ 4. $\cos(A+B) - \cos(A-B) = -2 \sin(A) \sin(B)$	P. 209-212	P. 156	Appendix A15
5. Apply the Sum and Product formulae to prove identities and to solve equations.		P. 213-214 P. 215-217	P. 156-158	Appendix A17

Additional References

1. Anton, H., Calculus With Analytic Geometry, (Brief Edition), John Wiley and Sons, Toronto, 1984.
ISBN 0-471-09443-9
2. Coxford, A.F., and J.N. Payne, HBJ Advanced Mathematics:
A Preparation for Calculus, Harcourt Brace Jovanovich,
New York, 1978.
ISBN 0-07-82531-9
3. Welchons, A.M., and W.R. Krickenberger, Trigonometry With Tables,
Ginn and Co., Toronto, 1957.

D. THE CALCULUS OF TRIGONOMETRIC FUNCTIONS

(13% elective)

This unit will give students the opportunity to apply differentiation and integration to trigonometric functions and to their related applications.

This elective unit should only be done after the unit on Trigonometric Extension. The concepts covered there are essential to the material covered in this unit.

Prescribed Resources

1. "Calculus, Complex Numbers and Polar Coordinates"
Holt, Rinehart and Winston - 1980
2. "Math 31"
Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
1. Review trigonometric identities.			
2. Evaluate the limits of trigonometric functions.		P. 191- 193	P. 191- 193
3. Develop the derivative of the sine and cosine functions.	Derivation is encouraged but should not necessarily tested.	P. 193- 194	P. 193- 194
4. Apply the sine and cosine derivatives of related trigonometric functions utilizing the Chain, Power, Product and Quotient Rules.		P. 193- 196	P. 193- 196
5. Solve applied maxima and minima problems.	This is a natural cut-off point for a 7% elective unit.	P. 196- 201	P. 196- 201

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
6. Find the family of curves whose first derivative has been given.		P. 201-203	P. 201-203
7. Solve applied problems dealing with periodic or repetitive phenomena.	Simple harmonic motion is an example of periodic phenomena. This is a natural cut-off point for a 10% elective unit.	P. 204-206	P. 204-206
8. Determine the area between a curve and the x-axis for a given domain.		P. 206-209	P. 206-209
9. Determine the area between two curves over a given domain.		P. 206-209	P. 206-209

Additional References

1. Ayres, F.A., Schaum's Outline of Theory and Problems of Differential and Integral Calculus, McGraw-Hill, 1964.
(This book is out-of-print.)
2. Swokowski, E.W., Elements of Calculus With Analytic Geometry, Prindle, Weber and Schmidt, Boston, Massachusetts.
ISBN 0-87150-504-5.

E. SYSTEMS OF LINEAR EQUATIONS

(7% elective)

The content of this elective is an extension of the Mathematics 20 program to three equations with three variables and an application of matrices.

Prescribed Resources

1. "Vectors, Matrices and Algebraic Structures"
Holt, Rinehart and Winston - 1980
2. "Math 31"
Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
1. Maintain previous skills in algebraically solving systems of equations with two unknowns.			
2. Distinguish between inconsistent, dependent, and independent systems of linear equations.	These systems could be illustrated graphically.	P. 161- 170	P. 451- 461
3. Algebraically solve systems of equations with three unknowns.	Reference should be made to the solution set as the intersection or non-intersection of planes. The discussion should include dependent, independent and inconsistent systems involving homogeneous and non-homogeneous equations. It should be noted that homogeneous equations cannot be inconsistent.	P. 149- 158 P. 161- 173	P. 443- 464

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
4. Define the following terms: 4.1 real matrix 4.2 matrix of coefficients of a linear system 4.3 augmented matrix of the system 4.4 row (column) equivalent matrices 4.5 row-reduced echelon form of a matrix		P. 174-182	P. 464-473
5. Solve a system of linear equations by reducing its augmented matrix to row-reduced echelon form.		P. 182-193	P. 473-484

Additional References

1. Coleman, A.J., J.J. Del Grande, G.F.D. Duff, J.C. Egsgard, B.J. Kirby, Algebra (Elements of Modern Mathematics), Gage Publishing Ltd., Toronto.
ISBN 0-7715-3636-4
2. Coxford, A.F., and J.N. Payne, HBJ Advanced Mathematics: A Preparation for Calculus, Harcourt Brace Jovanovich, New York, 1978.
ISBN 0-15-353940-2

F. MATRICES AND LINEAR TRANSFORMATIONS

(13% elective)

Special Note: Teachers selecting this unit might consider doing the elective on Systems of Equations first.

Matrices and linear transformations are not directly related to calculus or vectors but this topic is considered to be useful material for students who plan to take post secondary mathematics courses. The content may also be used in conjunction with computer programming.

Prescribed Resource

"Math 31"

Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.
		1984 Edition
1. Define an $m \times n$ real matrix.		P. 375-376
2. Define the dimensions of a matrix.		P. 375
3. Establish a convention to determine the position of entries in a matrix.	Example: $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \text{ OR } \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix}$	P. 375-376
4. Define the following with respect to matrices: 4.1 equivalent matrices 4.2 addition of matrices 4.3 multiplication by a real number 4.4 commutative property of addition 4.5 associative property of addition 4.6 additive inverse 4.7 additive identity		P. 377-382

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.
		1984 Edition
5. Solve problems using properties of matrices.		P. 379-380 P. 382-383
6. Define matrix multiplication.	Discuss the dimensional limitations of matrix multiplication.	P. 401-410
7. For matrix multiplication investigate the following algebraic properties: 7.1 closure 7.2 commutative 7.3 associative 7.4 multiplicative identity 7.5 distributive 7.6 divisors of zero	An example of divisors of zero: If $A = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix}$ then $\begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} -3 & 6 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ If $A \times B = 0$, then A or $B = 0$ does not necessarily apply.	P. 411-414
8. Define a linear transformation.		P. 384
9. Distinguish between onto and into transformations.		P. 386-389
10. Determine the matrix associated with a given transformation.		P. 384-385
11. Find the image of a given point under a given transformation by using matrix multiplication.	Example: If $T \begin{cases} u = 3x-y \\ v = 5x+2y \end{cases}$, then the image of $(-1, 2)$ under T is given by $\begin{bmatrix} 3 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$	P. 387-389

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.
		1984 Edition
12. Define the inverse of an onto transformation.	An into transformation does not have an inverse.	P. 393-395
13. Determine the conditions for which a transformation has an inverse.	Example: $T = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has an inverse if and only $ad-bc \neq 0$	P. 395
14. Define the product of two or more transformations.		P. 396-399
15. Solve problems related to special transformations.	Do problems dealing with rotations, magnifications, projections, reflections, etc.	P. 432-440

Additional References

1. Coleman, A.J., J.J. Del Grande, G.F.D. Duff, J.C. Egsgard, B.J. Kirby, Algebra (Elements of Modern Mathematics), Gage Publishing Ltd., Toronto.
ISBN 0-7715-3636-4
2. Coxford, A.F., and J.N. Payne, HBJ Advanced Mathematics: A Preparation for Calculus, Harcourt Brace Jovanovich, New York, 1978.
ISBN 0-15-353940-2
3. Crosswhite, F.J., L.D. Hawkinson, L. Sachs, Pre-Calculus Mathematics, Charles E. Merrill, Toronto.
ISBN 0-675-07931-4
4. Ebos, F., and B. Tuck, Math Is /4, Teacher's Edition, Nelson Canada, A Division of International Thomson Ltd., 1982.
ISBN 0-17-601414-4
5. Elliott, H.A., K.D. Fryer, J.C. Gardner, Norman J. Hill, Vectors and Matrices, Holt, Rinehart and Winston of Canada Limited, 1967.
6. Elliott, H.A., K.D. Fryer, J.C. Gardner, Norman J. Hill, Relations, Transformations and Statistics, Holt, Rinehart and Winston of Canada Limited, 1980. ISBN 0-03-921050-2.

G. POLAR COORDINATES

(7% elective)

Special Note: This elective should be selected only if the students have previously covered the trigonometry unit in Mathematics 30.

This unit will give the student the opportunity to deal with a different way of presenting an equation, rather than with the Cartesian coordinate plane. It should be pointed out that a complicated equation in the Cartesian system becomes a simple expression in the polar system.

Prescribed Resources

1. "Calculus, Complex Numbers and Polar Coordinates"
Holt, Rinehart and Winston - 1980
2. "Math 31"
Holt, Rinehart and Winston - 1984

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
1. Maintain previous skills developed for the Cartesian coordinate plane.			
2. Define the following terms: 2.1 pole 2.2 polar axis 2.3 radius vector 2.4 angle of rotation 2.5 polar coordinates	Stress both degree and radian measure for angles.	P. 241- 242	P. 241- 242
3. Plot points on a polar coordinate grid.		P. 243- 244	P. 243- 244
4. Draw graphs for functions given by simple polar equations.	Examples: 1. $r = 3 \cos (\theta)$ 2. $r = 2 \tan (\theta)$	P. 247- 251	P. 247- 251

OBJECTIVE	COMMENT/APPLICATION	TEXT REF.	
		1980 Edition	1984 Edition
5. Change rectangular coordinates to polar coordinates and vice versa.		P. 245-247	P. 245-247
6. Change the rectangular equation of a relation to the polar equation and vice versa.	A review of appropriate trigonometric identities may be necessary.	P. 245-247	P. 245-247
7. Graph polar relations.	Example: On St. Valentine's Day draw $r = 3(1 - \sin \theta)$. A discussion of symmetry about the polar axis, the pole, and the line $\theta = \frac{\pi}{2}$ would enhance the understanding of the graphs.	P. 247-251	P. 247-251

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APPENDICES



APPENDIX A

STRATEGIES OF PROBLEM-SOLVING

In the teaching/learning of problem-solving, an instructional approach should be used which helps students learn and choose procedures for solving problems. These procedures are easy to state and recognize, but they are often quite elusive when teaching. Difficulty frequently exists when teaching problem-solving because, unlike the teaching of computational skills or concepts, there is no specific content involved. In problem-solving, an individually acquired set of processes is brought to bear on a situation that confronts the individual.

There are generally four procedures (stages) which appear inherent in problem-solving. These procedures, their descriptions and associated strategies have been compiled and adapted from a variety of sources and authors (George Polya, J.F. LeBlanc, Ohio Department of Education, Math Resource Project, 1980, NCTM Yearbook) and are listed below:

STAGES IN PROBLEM-SOLVING

1. Understand the Problem

What is the problem? What are you trying to find? What is happening? What are you asked to do?

Suggested Strategies:

- Paraphrase the problem or question (Restate the problem in your own words to internalize what the problem entails.)
- Identify wanted, given and needed information (Helps students focus on what is yet to be determined from the problem statement as well as list information so that they may better be able to discover a relationship between what is known and what is required.)
- Make a drawing (May help to depict the information of a problem, especially situations involving geometric ideas.)
- Simulate situation (Helps to picture how the problem actions occur and how they are related thereby giving a better understanding of the problem.)
- Check for hidden assumptions (What precisely does the problem say or not say? Are you assuming something that may not be implied? Beware of mistaken inferences.)

2) Devise a Plan to Solve the Problem

What operations should you use? What do you need to do to solve the problem? How can you obtain more information or data to seek the solution?

Suggested Strategies:

- Solve a simpler (or similar) problem (Momentarily set aside the original problem to work on a simpler or similar case. Hopefully the relationship of the simpler problem will point to the solution for the original problem.)
- Construct a table (Organizing data in tabular form makes it easier to establish patterns and to identify information which is missing.)
- Look for a pattern or trend (Does a pattern continue or exist? In connection with the use of a table, graph, etc., patterns or trends may be more apparent.)
- Solve part of the problem (Sometimes a series of actions each dependent upon the preceding one, is required to reach a solution. Similarly, it may be that certain initial actions will either produce a solution or uncover additional information to simplify the task of solving the problem.)
- Make a graph or numberline (May help organize information in such a way that it makes the relationship between given information and desired solution more apparent.)
- Make a diagram or model (When using the model strategy attempt to select objects or actions to model those from the actual problem that represents the situation accurately and enables you to relate the simplified problem to the actual problem. May be used in connection with, or in place of other similar strategies, i.e., acting out the problem.)
- Guess and check (Guessing for a solution should not be associated with aimless casting about for an answer. The key element to this strategy is the "and check" when the problem solver checks his guesses against the problem conditions to determine how to improve his guess. This process is repeated until the answer appears reasonable. An advantage of this "guess and check" strategy is that it gets the individual involved in finding a solution by establishing a starting point from which he can progress. Used constructively with a table or graph this strategy may be a valuable tool.)

- Work backwards (Frequently, problems are posed in which the final conditions of an action are given and a condition is asked for which occurred earlier or which caused the final outcome. Under these circumstances working backwards may be valuable.)
- Change your point of view (Some problems require a different point of view to be taken. Often one tends to have a "mind set" or certain perspective of the problem which creates a difficulty in discovering a solution. Frequently, if the first plan adopted is not successful, the tendency is to return to the same point of view and adopt a new plan. This may be productive, but might also result in continuous failure to obtain a solution. Attempt to discard previous notions of the problem and try to redefine the problem in a completely different way.)
- Write an equation (Often in conjunction with other strategies - using a table, diagram, etc., one selects appropriate notation and attempts to represent a relationship between given and sought information in an open sentence.)

3. Carry Out the Plan

For some students the strategy/strategies selected may not lend itself/themselves to a solution. If the plan does not work, the problem-solver should revise the plan, review Stage 1, and/or try another plan or combination of plans from Stage 2.

4. Look Back At What Has Been Done (Consolidating Gains)

Is the result reasonable and correct? Is there another method or solution? Is there another solution? Is obtaining the answer the end of the problem?

- Generalize (Obtaining an answer is not necessarily the end of a problem. Re-examination of the problem, the result and the way it was obtained will frequently generate insights far more significant than the answer to the specific situation. It may enable the student to solve whole classes of similar and even more difficult problems.)
- Check the solution (The very length of a problem or the fact that symbolic notation is used may tend to make one lose sight of the original problem. Does the answer appear reasonable, does it satisfy all the problem requirements?)
- Find another way to solve it (Can you find a better way to confront and deal with the problem? The goal of problem-solving is to study the processes that lead to solutions to problems. Once a solution is discovered, search the problem for further insights and unsuspected ideas and relationships.)

- Find another solution (Students tend to approach many problem situations with the expectation of only one correct solution. In many practical, daily life situations there may be many answers that are correct and acceptable.)
- Study the solution process (Studying the process of solution makes the activity of problem-solving more than answer-getting and can expand an individual problem into a meaningful total view of a family of related problems.)
- Communicate your solution to others (Solutions should be stated in a clear and precise manner. Solutions should include units and other pertinent information from the problem.)

It must be noted that the four stages of the above model are not necessarily discreet. For example, one may move without notice into Stage 2 while attempting to generate more information to understand the problem better.

If the 4-stage model is used, the key is to select an appropriate strategy or strategies to help answer the questions suggested by each stage. The strategies listed, and those devised by students will hopefully alter the problem information, organize it, expand it, and make it more easily understood. Strategies then may be thought of as the tools of problem solving and the 4-stage model, the blueprint.

APPENDIX B

RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980s

The National Council of Teachers of Mathematics recommends that -

1. problem solving be the focus of school mathematics in the 1980s;
2. basic skills in mathematics be defined to encompass more than computational facility;
3. mathematics programs take full advantage of the power of calculators and computers at all grade levels;
4. stringent standards of both effectiveness and efficiency be applied to the teaching of mathematics;
5. the success of mathematics programs and student learning be evaluated by a wider range of measures than conventional testing;
6. more mathematics study be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population;
7. mathematics teachers demand of themselves and their colleagues a high level of professionalism;
8. public support for mathematics instruction be raised to a level commensurate with the importance of mathematical understanding to individuals and society.

APPENDIX C

ADDITIONAL REFERENCES

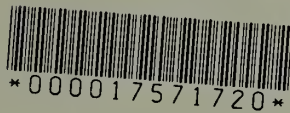
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Anton, H., <u>Calculus With Analytic Geometry</u> , (Brief Edition), John Wiley and Sons, Toronto. ISBN 0-471-09443-9	Calculus Calculus of Logarithmic Functions Polar Coordinates Trigonometric Extension	Conic Sections
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Boisen, M.D., and M.D. Larsen, <u>Understanding Basic Calculus With Applications From Managerial, Social and Life Sciences</u> , Charles E. Merrill Toronto. ISBN 0-675-08430-X	Calculus Applications of Calculus	
Coleman, A.J., J.J. Del Grande, G.F.D. Duff, J.C. Egsgard, B.J. Kirby, <u>Algebra</u> (Elements of Modern Mathematics), Gage Publishing Ltd., Toronto, 1979. ISBN 0-7715-3636-4	Vectors Systems of Linear Equations Matrices and Linear Transformation	Permutations and Combinations Mathematical Induction Probability Complex Numbers
Coxford, A.F., and J.N. Payne, <u>Advanced Mathematics: A Preparation for Calculus</u> , Harcourt Brace Jovanovich, New York, 1978. ISBN 0-15-353940-2	Trigonometric Extension Matrices and Linear Transformation Introduction to Calculus Systems of Linear Equation Volumes of Revolution Polar Coordinates	Trigonometry Sequences, Series Exponential and Logarithmic Functions Conic Sections

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Crosswhite, F.J., L.D. Hawkinson, L. Sachs, <u>Pre-Calculus Mathematics</u> , Charles E. Merrill, Toronto. ISBN 0-675-07931-4	Vectors in the Plane Polar Coordinates Logarithms and Exponentials Limits Derivations Integrals Matrices and Linear Transformation	Polynomial Functions Conics Trigonometric Functions Functions Coordinate Geometry Number Systems Graphing
Del Grande, J.J. and G.F.D. Duff, <u>Calculus</u> , 2nd Edition, Gage Publishing Ltd., Toronto. ISBN 0-7715-3634-8	Reference to Calculus Polar Coordinates	
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Lynch, R.O., <u>A First Course in Calculus Including Analytic Geometry</u> , 3rd Edition, Ginn and Company, Lexington, Massachusetts. ISBN 0-663-41967-0	Derivations and Applications of Derivatives Integration Limits Logarithmic and Exponential Functions Polar Coordinates Volumes of Revolution	Graphs - Curve tracing
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